A Three-Flavor AdS/QCD Model with a Back-Reacted Geometry

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Abstract

A fully back-reaction geometry model of AdS/QCD including the strange quark is described. We find that with the inclusion of the strange quark the impact on the metric is very small and the final predictions are changed only negligibly.

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I. INTRODUCTION

QCD [1] is considered to be a well-established theory for the strong interaction. In the high energy regime, we can use a perturbative approach to understand the theory. However, at low energy, because of the large coupling constant, perturbation theory is not applicable. In the low energy regime we can appeal to other methods of analysis, for instance chiral perturbation theory and lattice QCD.

Conjectured by Maldacena [2] in 1997, the AdS/CFT correspondence is a new approach to this difficult problem. This conjecture states that a string theory on $AdS_5 \times S^5$ is equivalent to a conformal theory on the boundary of AdS_5. QCD is classically but not quantum mechanically conformal. However, the AdS/CFT correspondence has provided important insights into QCD, such as confinement at large distances [3] and chiral symmetry breaking[4], [5], [6], [7], [8], [9], [10], [11]. Currently these topics are very active areas of research.

The quantitative correspondence was specified in independent work by Gubser, Klebanov and Polyakov [12] and by Witten [13]

$$\left\langle e^{i \int d^4 x \, \mathcal{O} \phi} \right\rangle_{CFT} = \mathcal{Z}_{SUGRA} \left(\phi(z)|_{z \to 0} = \phi \right) \tag{1}$$

which states that the generating functional for correlation functions with a source ϕ for some field theory operator is equivalent to the partition function of a supergravity theory where the boundary value of some supergravity field is the source for the field theory operator. The choice of supergravity field and field theory operator is a matter of matching the representations of the global symmetries of the two pair.

In recent years a new phenomenological approach, based on the rules of the AdS/CFT correspondence has been developed [6],[8]. This approach introduces a five-dimensional classical theory in an AdS_5 background where appropriate fields are included in the action to act as sources on the boundary for operators of a QCD-like theory. This original formulation included only light quark operators and gave a phenomological model of chiral symmetry breaking. In the five-dimensional theory the symmetry is a gauge symmetry and a simple Higgs mechanism is set up to model chiral symmetry breaking in the four-dimensional theory.

The results from these relatively simple and phenomenological models are remarkable and the simplest realisation gives postdictions for several meson masses and decay constants with an average of around 15% error.

Since the introduction of this phenomenological action, many advances have been made to model QCD more accurately. These include the introduction of linear confinement via an appropriately chosen scalar field in the five-dimensional theory[14], the inclusion of gluon condensate contributions to QCD quantities[15] and studies of heavy quark potentials[16],[17],[18].

In [19], we considered the impact of a classical scalar field back-reacting on the geometry. In this case the impact on the geometry was most strongly affected by the condensate of light quarks.

The strange quark was introduced [11] in order to study the kaon sector and found that reasonably accurate predictions could be found for these mesons, too.

In this paper we ask what the impact of the strange quark on the geometry will be. We may expect that as the chiral symmetry is broken more explicitly for the strange quark the effect of the strange quark condensate on the dynamics of the theory may be less pronounced.

II.   BACK REACTION ON THE GEOMETRY

In this section, we consider the impact of one scalar field on the metric. The total field content is the gravitational field plus the scalar field, which will be responsible for chiral symmetry breaking. The Lagrangian is given by

$$S = \int d^5x \sqrt{g}(-R + Tr(\partial\phi)^2 + V(\phi)), \tag{2}$$

R is the five-dimensional Ricci scalar, and the metric is

$$ds^2 = e^{-2A(y)}dx_\mu dx^\mu - dy^2. \tag{3}$$

The Ricci scalar R is given by

$$R(y) = 20A'^2(y) - 8A''(y) . \tag{4}$$

From the action, one can find the equations of motion for the scalar field and for the metric tensor.

$$\frac{1}{2}g_{PQ}[-R + Tr(\partial_M\phi\partial^M\phi + V(\phi))] + R_{PQ} - Tr\partial_P\phi\partial_Q\phi = 0 , \tag{5}$$

and

$$Tr\frac{\partial V(\phi)}{\partial\phi} = \frac{2}{\sqrt{g}}Tr\partial_P(\sqrt{g}g^{PQ}\partial_Q\phi) , \tag{6}$$

which gives

$$6A''(y) - 12A'^2(y) + Tr(V(\phi) - \phi'^2(y)) = 0 \tag{7}$$

$$12A'^2(y) - V(\phi) = Tr\phi'^2(y) \tag{8}$$

$$\phi''(y) + 4A'(y)\phi'(y) + \frac{1}{2}\frac{\partial V(\phi)}{\partial \phi} = 0. \tag{9}$$

Eq.(7) and Eq.(8) give

$$3A''(y) = Tr\phi'^2(y) \tag{10}$$

and

$$3A''(y) - 12A'^2(y) + V(\phi) = 0 . \tag{11}$$

From Eq.(10), the function of $A(y)$ can be obtained, given a solution for ϕ. Then from Eq.(11), one can find the potential $V(\phi)$. So, at no point do we need to rely on numerical techniques.

We now give an example and show how to find the warp factor in the metric function in the presence of a scalar field. Consider a scalar field, given by

$$\phi(y) = \frac{m_q}{2}e^y + \frac{\sigma}{2}e^{3y} , \tag{12}$$

where m_q and σ are 3 by 3 matrix.

$$m_q = diag(m, m, m_s), \quad \sigma = diag(c, c, c_s).$$

We find

$$Tr\phi'^2(y) = 2\left(\frac{3}{2}ce^{3y} + \frac{1}{2}me^y\right)^2 + \left(\frac{3}{2}c_se^{3y} + \frac{1}{2}m_se^y\right)^2. \tag{13}$$

Then from Eq.(10), and the UV boundary condition $A'(y)_{y\to-\infty} = 1$, the warp factor $A(y)$ is found to be

$$A(y) = y + \frac{1}{8}\left(\frac{1}{3}c^2e^{6y} + \frac{1}{6}c_s^2e^{6y} + \frac{1}{2}ce^{4y} + \frac{1}{4}c_se^{3y}m_s\right) . \tag{14}$$

We see that the UV behaviour of the metric is not greatly modified by the back reaction of this scalar field.

III.   A PHENOMENOLOGICAL MODEL

In this model of QCD, the three relevant operators are $\bar{q}_L^\alpha q_R^\beta$, $\bar{q}_{L,R}\gamma^\mu t^a q_{L,R}$.
Now, let's consider the following action

$$S = \int d^5x\sqrt{g}\left\{-R + Tr\left(|D\phi|^2 + V(\phi) - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right)\right\} \tag{15}$$

where $D_\mu\phi = \partial_\mu\phi - iA_{L\mu}\phi + i\phi A_{R\mu}$, $A_{L,R} = A^a_{L,R}t^a$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$. We define the vector and axial-vector gauge bosons to be $V_M = \frac{1}{2}(L_M + R_M)$ and $A_M = \frac{1}{2}(L_M - R_M)$ respectively. Following [6] we choose the $V_z = A_z = 0$ gauge. From this action the classical value of the scalar field is found to be that we chose in Eq.12. Substituting $\phi = <\phi> e^{i2t^a\pi^a(x,y)}$ back into the action, the mass matrix of V_M and A_M bosons can be calculated [11].

$$M_V^2 = \begin{pmatrix} \mathbf{0_{3\times3}} & 0 & 0 \\ 0 & \frac{1}{4}(\hat{m} - \hat{m}_s)^2 z^2 \mathbf{1_{4\times4}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{16}$$

and

$$M_A^2 = \begin{pmatrix} \hat{m}^2 z^2 \mathbf{1_{3\times3}} & 0 & 0 \\ 0 & \frac{1}{4}(\hat{m} + \hat{m}_s)^2 z^2 \mathbf{1_{4\times4}} & 0 \\ 0 & 0 & \frac{1}{3}\left((\hat{m})^2 + 2(\hat{m}_s)^2\right) z^2 \end{pmatrix}, \tag{17}$$

where $\hat{m} = m + cz^2$ and $\hat{m}_s = m_s + c_s z^2$. The equations of motion for the vector and axial vector bosons can also be derived from Eq.(15). For convenience we make the following change of variable $z = e^y$.

$$\left[\partial_z^2 + \partial_z(\ln a)\partial_z + \left(q^2 - (g_5^2 a^2 M_V^2)_{\alpha\alpha}\right)\right]\Phi_V^\alpha(q, z) = 0, \tag{18}$$

and

$$\left[\partial_z^2 + \partial_z(\ln a)\partial_z + \left(q^2 - (g_5^2 a^2 M_A^2)_{\alpha\alpha}\right)\right]\Phi_A^\alpha(q, z) = 0, \tag{19}$$

where $a = a(z, m, c, m_s, c_s) = \frac{1}{z}exp\left(-\frac{1}{8}(\frac{1}{3}c^2 z^6 + \frac{1}{6}c_s^2 z^6 + \frac{1}{2}cz^4 m + \frac{1}{4}c_s z^4 m_s)\right)$, with boundary conditions $\partial_z\Phi_V^\alpha(q, z_{IR}) = 0$, and $\Phi_V^\alpha(q, \epsilon) = 0$, similarly for Φ_A^α. The mass of the vector and axial vector mesons can be obtained by solving the eigenvalue equations Eq.(18) and Eq.(19) with $q^2 = m_V^2$ and $q^2 = m_A^2$, respectively. The decay constants of these mesons can be obtained by

$$F_{V,A}^2 = \frac{1}{g_5^2}\left(\frac{\Phi_{V,A}''(0)}{N}\right)^2, \tag{20}$$

where

$$N = \int_0^{z_{IR}} dza|\Phi_{V,A}(z)|^2.$$

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and the mass of the pseudoscalar meson can be obtained by solving

$$\left(\partial_z^2 + \partial_z \left(\ln a\right) \partial_z\right) \phi^a + g_5^2 a^2 \left(M_A^2\right)_{\alpha\alpha} (\pi^\alpha - \phi^\alpha) = 0,$$

$$\partial_z \left(a^3 \left(M_V^2 + M_A^2\right)_{\alpha\alpha} \partial_z \pi^\alpha\right) = q^2 a^3 \left(\left(M_V^2 + M_A^2\right)_{\alpha\alpha} \left(\frac{1}{2}\phi^\alpha - \pi^\alpha\right) + \left(M_A^2 - M_V^2\right)_{\alpha\alpha} \frac{1}{2}\phi^\alpha\right),$$

$$(21)$$

with boundary conditions:

$\partial_z \phi^\alpha(z = z_{IR}) = \partial_z \pi^\alpha(z = z_{IR}) = \phi^\alpha(z = 0) = \pi^\alpha(z = 0) = 0$, where ϕ^α is defined as the longitudinal part of A_μ^α, $\partial_\mu \phi^\alpha = A_{\mu||}^\alpha$.

The decay constants of the pseudoscalar

$$f_{P\alpha} = -\frac{1}{g_5^2} \frac{\partial_z A^\alpha(0, z)}{z}\Big|_{z=\epsilon},$$

$$(22)$$

with $A^\alpha(0, z)$ are given by the solution of Eq.(19) satisfying $A^{\alpha\prime}(0, z_{IR}) = 0$ and $A^\alpha(0, \epsilon) = 1$.

In order to generate a mass gap, we need to introduce an IR cutoff(z_{IR}). The fifth dimension is taken as an interval from 0 to z_{IR}.

The model now has six free parameters: g_5^2, z_{IR}, m, c, m_s, c_s. g_5^2 can be obtained by comparing the vector-vector two point function obtained from the OPE of QCD to that obtained using the holographic recipe[6], giving $g_5^2 = \frac{N_c}{12\pi^2}$. Thus there are five free parameters left, we use an iterative method to fit the five free parameters.

We start by fitting the parameters without the back reaction. That is, as a starting point we choose $a(z, m, c, m_s, c_s) = \frac{1}{z}$. Using the following experimental data: $m_\pi = 139.6 MeV, f_\pi = 92.4 MeV, m_\rho = 775.8 MeV, m_{K1A} = 1339 MeV$, and a semi-global fit for m_{K^*}, we then use an iterative search method to fix the free parameters in order to minimise the rms error on the remaining data. The final fit results are shown in Table I. Having fixed the free parameters, we can calculate the remaining mesons masses and decay constants. In Table II and Table III, we show the mass and decay constants of vector mesons and axial vector mesons respectively.

TABLE I: Fit results for the free parameters in units of MeV.

z_{IR}^{-1}	m	$c^{\frac{1}{3}}$	m_s	$c_s^{\frac{1}{3}}$
320.55	2.28	328.5	138.5	176

TABLE II: Axial vector meson results calculated with a back reacted geometry and the free parameters given in Table I. ∗ indicates that this value is used to fix the free parameters, all other values are predictions. Numbers in brackets give the percentage error.

observation	$value(MeV)(\%error)$
m_π	139.6*
f_π	92.4*
m_{a1}	1364(10.9)
$\sqrt{F_{a1}}$	440(1.6)
$m_{K_{1A}}$	1339*
$\sqrt{F_{K_{1A}}}$	435(4.1)
m_{A3}	1344
$\sqrt{F_{K_{A_3}}}$	412

TABLE III: Axial vector meson results calculated with a back reacted geometry and the free parameters given in Table I. ∗ indicates that this value is used to fix the free parameters, all other values are predictions. Numbers in brackets give the percentage error.

observation	$value(MeV)(\%error)$
m_ρ	775.8*
$\sqrt{F_\rho}$	348.8(1.1)*
$m_{\rho'}$	1781
$\sqrt{F_{\rho'}}$	658
m_{K^*}	812(9)
$\sqrt{F_{K^*}}$	328(11)
m_{V_3}	m_ρ
$\sqrt{F_{V_3}}$	$\sqrt{F_\rho}$

Having fitted the free parameters and calculated the remaining meson properties we can also

calculate the Ricci scalar for the back reacted geometry:

$$R(z) = -12c^2z^6 - 6c_s^2z^6 - 8cz^4m - 4c_sz^4m_s + \frac{5}{16}\left(8 + 2c^2z^6 + c_s^2z^6 + 2cz^4m + c_sz^4m_s\right)^2. \quad (23)$$
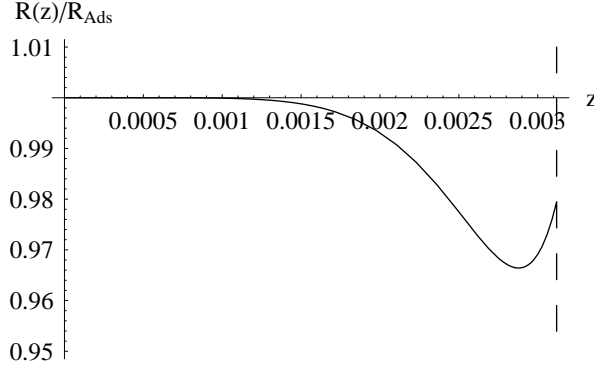

FIG. 1: The variation of the Ricci scalar as a function of radial distance in our model with with parameters in Table I.

We plot the curvature as a function of the radial distance in the AdS space, see Figure 1. Figure 1 shows that the back reaction has only a small impact on the scalar curvature in the interval $(0, z_{IR})$ with a maximum of around 3% departure from the pure AdS result. For $z > z_{IR}$ the impact is larger but has no effect on our results.

We also calculate the mass of ρ resonances which is shown in Figure 2. However, because of the small difference between no-back-reaction case and back-reaction case, the two lines are almost indistinguishable on this scale. As the stringy effects are neglected in our present analysis, they are expected to become important in the UV. Thus, the reliability of the current models will diminish above the scale of chiral symmetry breaking (around 1200 MeV).

The main conclusion of this calculation is that even with the addition of strange quark dynamics the geometry and hence the spectra of masses and decay constants are not heavily affected. This is a non-trivial statement about the impact of the strange quark on chiral dynamics.
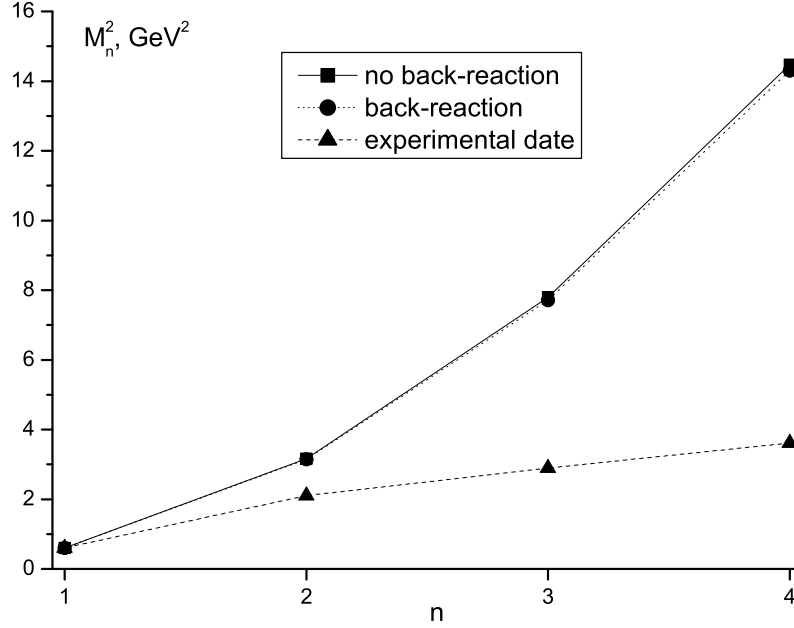
FIG. 2: Experimental values for $M_{\rho^*}^2(GeV^2)$ against theoretical values both with and without back-reaction. The results without back-reaction are calculated with $z_{IR}^{-1} = 322.6 MeV$ and those with back-reaction are calculated using the parameters given in Table I.

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